PARTICULARITIES ON NUMERICAL MODELING OF CRUCIBLE INDUCTION FURNACE

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The paper presents some particular aspects regarding numerical modeling of crucible induction furnaces. The problem is treated as a coupled magneto-thermal application. The conditions for an accurate FEM analysis of this device are determined. Some models for material properties and solutions for the thermal problem of the covered crucible furnace are proposed. A complex concrete application emphasizes these considerations.

1. INTRODUCTION

Crucible induction furnace consists mainly of a crucible made by refractory material, in which the metallic mass to be heated and melted is located, a thin layer of thermal insulant material and an induction coil cooled by water circulation. Other parts are the yokes, placed around the coil at regular intervals, the metallic frame, a tilting device and, in most cases, a lid [1].

Even if crucible induction furnace is not a recent induction heating application, the continuous development of numerical techniques and increasing of the computers performances give the possibility of an advanced analyse of this device. The purpose is to elaborate the numerical model of the furnace. Having the model, an optimal synthesis is allowed in order to improve their operating parameters (e.g. efficiency, specific energy consumption and stirring) by the means of correlation between geometric parameters, power supply and materials properties. In this paper the FEM analysis of electromagnetic and thermal process within crucible induction furnace is performed. Flux 2D professional package was used as a support for numerical modeling [2].

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2. MAGNETO-THERMAL MODELING OF THE CRUCIBLE INDUCTION FURNACE

2.1. THE MAGNETODYNAMIC PROBLEM

In accordance with § 1, we propose the calculus domain shown in figure 1, in $rOz$ coordinates. This 2D structure corresponds to half axial section of the physical model represented in cylindrical coordinates.

The subregions are: 1 – the charge – conducting region, magnetic or nonmagnetic, function of metal composition; 2 – the refractory lining – nonmagnetic and nonconductive region; 3 – thermal insulant material (if exists) – nonmagnetic and nonconductive region; 4 – a thin thermal insulant layer (between the crucible and the inductor, for thermal protection of coil windings) – nonmagnetic and nonconductive region; 5 – inductor - conductive and nonmagnetic region (usually, copper), with or without induced currents as it follows:

– if in magnetic problem the inductor is considered made by thin wire, the induced currents are zero or negligible; this non-resistive region has vacuum properties and the current density (source) is constant in the inductor cross-section;
– if in magnetic problem the inductor is considered made by solid conductor (having the number of windings, dimensions of a winding section, stacking factor etc.), there are produced the induced currents; the resistive region has copper properties and the source is the potential between the ends of the solid conductor.

6 – surrounding (air) – nonmagnetic and nonconductive region; 7 – magnetic yokes – may be considered magnetic and nonconductive region, because of low level of specific losses within the laminations. There are two exceptions: first, the furnaces operating at high frequencies when it is necessary to consider the yokes as a magnetic and conductive region because of increased losses (the specific methods for modeling the laminations are developed) and second, the special case of low capacity furnaces, operating at tens or hundreds kHz, when the shields are made by copper and we must consider this region as a nonmagnetic and conductive one.

Because of the symmetry, the modeling of the crucible induction furnace is a 2D axisymmetric problem. The currents have only orthoradial components (on $\phi$); the magnetic vector potential has the same direction as the current. The mathematical model expressed in magnetic vector potential $A$ is given by the equation:

$$\text{rot} \left[ \frac{1}{\mu} \text{rot} A \right] + j \frac{\omega}{\rho} A = J_1$$  \hspace{1cm} (1)

where, in general case, we have the resistivity $\rho = 1/\sigma = \rho(\theta)$ variable with temperature and the magnetic permeability $\mu = 1/\nu = \mu(\theta, H)$ variable with temperature and magnetic field, respectively the reluctivity, $\nu = \nu(A, \theta)$.

In (1), the second term to the left represents the density of induced currents, null within a nonconductive region. If we consider an inductor made by thin wire, this term is non-null only in charge. The second member represents the current density source in inductor. We can write (1) for the regions of calculus domain:

– the charge (1):

$$\text{rot} \left[ (1/\mu) \text{rot} A \right] + j \omega \frac{1}{\rho} A = 0 \hspace{1cm} (2)$$

– the crucible layers and air (2+3+4+7):

$$\text{rot} \text{rot} A = 0 \hspace{1cm} (3)$$

– the inductor (5):

$$\text{rot} \text{rot} A = \mu_0 J_1 \hspace{1cm} (4)$$

– the yokes (6), if exists:

$$\text{rot} \left[ (1/\mu_{Fe}) \text{rot} A \right] = 0 \hspace{1cm} (5)$$

With respect to axisymmetric problem, the equation can be written in cylindrical coordinates ($r-z$ plane). The problem is solved using the modified magnetic potential $rA$. For a harmonic excitation, we have:

$$-\left[ \frac{\partial}{\partial r} \left( \frac{v}{r} \frac{\partial}{\partial r} (r \cdot A) \right) + \frac{\partial}{\partial z} \left( \frac{v}{r} \frac{\partial}{\partial z} (r \cdot A) \right) \right] + j \omega \frac{\sigma}{r} (r \cdot A) = J_{ex} \hspace{1cm} (6)$$
where $\sigma$ is electrical conductivity and $J_{ex}$ is current density source in the inductor. If the problem is voltage driven, the right-hand side term may become temperature dependent as well: $J_{ex} = \sigma(\theta) \cdot V_s$.

If we assume (in some cases) that the magnetic permeability $\mu = 1/\nu$ is constant, this 2D problem is much easier; Coulomb gauge is implicit satisfied and the vector potential is characterized by a single unknown value.

In the calculus domain shown in figure 1, the inductor region is presented in a simplified mode, as a rectangular region, in $r-z$ plane. The dimensions of this rectangle are: the penetration depth $\delta_1$ in the inductor copper, at the working frequency and the real height of the inductor. When we assign the source to this region, we must take into account the axial filling factor of the coil.

For the magnetic problem, the boundary conditions are represented in figure 3 and have the following explanations:

- $Oz$ axis is considered a magnetic field line, so $A = 0$, respectively $U = r \cdot A = 0$;
- all other lines that close the calculus domain are considered so far situated from strength field zone as these lines represents the infinity, where the magnetic field becomes null and $A = 0$, respectively $U = 0$.

2.2. THE THERMAL PROBLEM

Corresponding to calculus domain for the magnetic field, the calculus domain for the transient thermal field for the crucible induction furnace is shown in fig. 2. The regions for this domain are: 1 – the charge, 2 – the refractory lining, 3 – the thermal insulation of the crucible and 4 – the thermal insulant thin layer between the crucible and the inductor. These regions represent materials with thermal properties known from the initial data.

The mathematical model of the transient thermal field $\theta = \theta (r, z, t)$ is expressed by the equation:

$$\left( \gamma C_p \right) \frac{\partial \theta}{\partial t} + \operatorname{div} \left( -\lambda \ \operatorname{grad} \theta \right) = p_j$$

where $\gamma$ is the density, $C_p$ is the specific heat, $(\gamma C_p)$ is the calorific capacity and $\lambda$ is the thermal conductivity. The right side member of (7) represents the source of the thermal field, the volumic density (mean value in a period) of the electromagnetic power developed by Joule effect of the eddy currents:

$$p_j = \frac{1}{2} \sigma \omega^2 AA^*$$

This is the main coupling element between the electromagnetic and thermal phenomena.

For the uniqueness of solution of equation (7) it is necessary to know the initial thermal field and the boundary condition, besides the thermal field source
and the materials properties. The initial condition associated to model of transient thermal field is $\theta(r, z, 0) = \theta_0$, that means that, on the beginning of the heating process, all device is at the surrounding temperature $\theta_0$, an acceptable hypothesis for an tap and charge functioning furnace. For other situations it is possible to start the process at other temperatures $\theta_0 \neq \theta_a$.

The boundaries of the calculus domain are in fact the surfaces through that the thermal transfer from the furnace parts to the surrounding or cooling water is done. The boundary conditions satisfied by unknown $\theta(r, z, t)$ at anytime are:

- at the symmetry axis ($Oz$), respectively the symmetry axis of the thermal field, $\partial \theta / \partial n = 0$, that means a null thermal flux condition;
- at all surfaces of thermal exchange (convection and radiation) with the exterior:

$$- \lambda \frac{\partial \theta}{\partial n} = \alpha_{\text{conv}}(\theta - \theta_a) + \varepsilon_{\text{rad}} \cdot C_n \cdot \left(\theta^4 - \theta_a^4\right)$$

where on the right side, the first term represents the convection and the second term, the radiation. $\alpha_{\text{conv}}$ is the convection coefficient, $\varepsilon_{\text{rad}}$ is the total emissivity and $C_n = 5.67 \cdot 10^{-8}$ W/m$^2$K$^4$ is Stefan constant.

The coefficients $\alpha_{\text{conv}}$ and $\varepsilon_{\text{rad}}$ could depend on temperature and $\varepsilon_{\text{rad}}$ could depend in addition on surface quality. To notice that for the charge surface $\varepsilon_{\text{rad}}$ has a strong variation from the initial state (the charge may be made of oxidized metallic wastes) to final state (the surface of the metallic bath).

A particular case is represented by thermal exchange surface between the inductor and the cooling water that circulates through the windings. Here thermal transmission is realised only by convection and so there is only one term on the right side of the equation (9):

$$- \lambda \frac{\partial \theta}{\partial n} = \alpha_{\text{conv ind--apa}}(\theta - \theta_{\text{med apa}})$$

where $\alpha_{\text{conv ind--apa}}$ is the transmissivity between the inductor and cooling water whose medium temperature is $\theta_{\text{med apa}}$.

Usually, in the numerical model, this condition is imposed to the surface between the thermal insulation layer and the inductor region (which in these situations doesn’t appear in the thermal problem); this condition can be also imposed to the surface of separation between the yokes and inductor only if we consider that the yokes are stuck to the inductor (as it is shown in fig. 2). This way of imposing boundary conditions to the surfaces between the inductor and nearly (solids) regions does not give significant errors because the high thermal conductivity of copper (about 393 W/mK) determines a negligible temperature decrease through the small thickness of the inductor winding.
2.3. THE COUPLING

These models are used to solve the magneto-thermal problem of the crucible induction furnace, respectively to study the coupling between the harmonic electromagnetic field represented by the unknown $A(r,z)$ and transient thermal field $\theta(r,z,t)$ in the calculus domain. It is possible to solve this coupled problem in different ways, the most used method being an alternate coupling method (step by step). So, at the beginning of the process ($t = 0$) in a 2D system it is determined the distribution of the harmonic electromagnetic field. Based upon this and knowing the values of the electromagnetic properties of the materials, corresponding to the initial temperature, it is determined the volumic density of the induced power in charge (or in charge and conductive crucible, in cases of steel or graphite crucible [3]).

![Coupling algorithm – the alternating coupling method](image)

Having the distribution of the thermal sources we can make a first determination of the distribution of thermal field at the first time step. Corresponding to these temperature values we can update the physical properties of the problem (material properties and thermal transfer characteristics). Then, we go through an iterative scheme *calculus of electromagnetic field* – *calculus of thermal field* – *updating properties* finally resulting the solution of coupling between electromagnetic and thermal fields at first time step. Next, there is a sequence of
time steps in the same way, the process being stopped by the condition of reaching a value of temperature. For the studied process, the end of the heating corresponds at pouring temperature. It is important to calculate the mean (in a charge volume) temperature because in this case we can considered that we take into account the uniformity of the temperature of the liquid charge due to stirring. The stirring is not studied in this paper

2.4. THE RESULTS

When we have the numerical model, it is possible to determine the distribution of electromagnetic and thermal fields in space and in time that means to determine the state variables, magnetic vector potential $A$, for electromagnetic field and temperature $\theta$, for thermal field, in each node of the finite element mesh and to know the time variation of these quantities. This fact allows calculating the important quantities that characterize the furnace: electrical and thermal efficiency, power factor, global efficiency of the entire process and the parameters of an equivalent scheme (total resistance and reactance).

We determine the energetic parameters as follow:

- thermal efficiency: $\eta_t = (P_{j2} - P_t) / P_{j2}$  \hspace{1cm} (11)
- electrical efficiency: $\eta_e = P_{j2} / (P_{j2} + P_{j1})$  \hspace{1cm} (12)
- total efficiency: $\eta = (P_{j2} - P_t) / (P_{j2} + P_{j1})$  \hspace{1cm} (13)

We consider above:

- the induced power in charge: $P_{j2} = \int_{V_2} \rho_2 J_2^2 dV$  \hspace{1cm} (14)
- Joule loses in the inductor: $P_{j1} = \int_{V_1} \rho_1 J_1^2 dV$  \hspace{1cm} (15)
- thermal loses $P_t$: $P_t = \sum_{i=1}^{n} A_i \int_{\Gamma_i} \alpha_i (\theta - \theta_a) ds$  \hspace{1cm} (16)

where $\rho_2$ is the resistivity of the metallic charge in each node of the mesh, $J_2$ is the local induced current density at the same points and $V_2$ is the charge volume. Because during the process it is a large variation of charge temperature (generally between the ambient temperature and melting or pouring temperature) it is an imperative condition to consider the variation of charge resistance with temperature for accurate results of modeling. It is noticed that these variation determine the time variation of induced power in charge $P_{j2}$ and as a consequence the variation of the efficiencies calculated as above during the process.
Similar, in (15), $\rho_1$ is the resistivity of inductor copper and $J_1$ is the current density in the inductor (source of electromagnetic field). Considering the mean temperature of the cooling water $\theta_{apa\ med} = 60 \ ^{\circ}C$, we obtain: $\rho_1 = 1.9 \cdot 10^{-8} \ \Omega m$.

Concerning the current density source in the inductor, it is possible to solve the problem in two different ways:

- first variant, - current supply - consists in imposing of a value for current density in the inductor, constant for entire process. This is the case of a constant current source (a converter which maintain the constant current whatever is the variation of the equivalent resistance and reactance of the furnace during the process);

- second variant - voltage supply – assume that the voltage applied to inductor is constant during the process. Knowing the configuration of the inductor (number of turns, dimensions of cross-sections, insulation) results a total current and a current density in the inductor. But these values are not constants because the total parameters of the furnace vary during the heating. In this case results the variation of the current and Joule loses $P_{J1}$ in the inductor.

In thermal loses $P_t$ (16) on the surfaces where thermal exchange with exterior of the furnace is possible, we consider $n$ boundaries $\Gamma_1, \Gamma_2, \ldots, \Gamma_n$, having surfaces $A_i$ and global exchange coefficients $\alpha_i$. The global exchange coefficient is in fact temperature dependent quantity because of temperature variation of the convection and radiation coefficients. Results a large variation of $P_t$ during the process.

As a consequence, the efficiencies (11-13) have a strongly variation during the heating process. Global efficiency, represents the quantity which characterize better the entire electro-thermal process within the furnace:

$$\eta_{global} = \frac{P_{12\ med} - P_{med}}{P_{12\ med} + P_{J1}}$$

(17)

where:

$$P_{12\ med} = \frac{\int_0^t \int_{V_2} \rho_2 J_2^2 \ dV \ dt}{t_1}$$

(18)

is the mean value of the induced power in charge during heating, and

$$P_{t\ med} = \frac{\int_0^t \left( \sum_{i=1}^n A_i \int_{\Gamma_i} \alpha_i (\theta - \theta_s) \ ds \right) dt}{t_1}$$

(19)

represents the mean value of the thermal losses.

In a special furnaces with conductive crucible (graphite or steel), used in some cases for melting nonferrous alloys or metals having a great conductivity or
precious metals, the heating is a mixed one – by induction and by thermal conduction from heated crucible. In this case, the overall efficiency is:

\[
\eta_{\text{global}} = \frac{P_{J,\text{med}} + P_{J,\text{grafit med}} - P_{J,\text{med}}}{P_{J,\text{med}} + P_{J,\text{grafit med}} + P_{J,\text{top}}} 
\]

(20)

where \( P_{J,\text{grafit med}} \) is the mean value of the induced power in the conductive crucible.

3. THE MATERIALS PROPERTIES

For an accurate magneto-thermal analysis of the crucible induction furnace it is necessary to know precisely the variation with temperature of materials properties and thermal exchange conditions for entire temperature variation domain. In addition, if we try to study the heating process above the melting point, it is important to take into account the variation of all parameters during the change of phase. We propose here some variation laws for materials properties and exchange conditions in order to simulate better the electrothermal process within crucible induction furnace.

For example, for cast-iron heated and melted in induction furnace, the models used for properties (variable with temperature) are:

- resistivity:
  \[
  \rho = 0,51 \cdot 10^{-6} \cdot (1 + 0.001 \cdot \theta) \ \Omega \text{m} \quad (21)
  \]

- thermal conductivity:
  \[
  \lambda = 2,5 \cdot 10^{-7} \cdot (\theta^3 - 0.05 \cdot \theta + 50) \ \text{W/mK} \quad (22)
  \]

- relative magnetic permeability, given implicit by the dependence
  \[
  B(H, \theta) = \mu_0 H + J_{s0} \cdot \frac{H_s + 1 - \sqrt{(H_s + 1)^2 - 4 \cdot H_s (1-a)}}{2 \cdot (1-a)} \cdot \text{coef(\theta_o)}
  \]
  where \( H_s = \mu_0 H \cdot \frac{\mu_0 - 1}{J_{s0}} \), \( \mu_0 \) is the initial relative magnetic permeability, \( J_{s0} \) is the saturation magnetization, a is a coefficient for adjusting the magnetization curve \((0 \leq a \leq 0.5)\) and coef(\theta) show the variation with temperature of the magnetic properties.

- specific heat – the model "gaussian + exponential" [2], must take into account both phase transition during entire process, at Curie point and at the melting point.

\[
\begin{align*}
\gamma_c &= \exp \left( -0.5 \left[ \frac{\theta - \theta_{\text{Curie}}}{\sigma_{\text{Curie}}} \right]^2 \right) + \left( V_0 - V_i \right) \exp \left( -\frac{\theta - \theta_{\text{Curie}}}{\tau_{\text{Curie}}} \right) + V_i, \text{ if } \theta \leq 1105 \ ^\circ \text{C} \\
\gamma_c &= \exp \left( -0.5 \left[ \frac{\theta - \theta_{\text{top}}}{\sigma_{\text{top}}} \right]^2 \right) + \gamma_c(1105) \exp \left( -\frac{\theta - \theta_{\text{top}}}{\tau_{\text{top}}} \right) + V_i, \text{ if } \theta > 1105 \ ^\circ \text{C}
\end{align*}
\]

(24)
where \( E_{\text{Curie}} \) and \( E_{\text{top}} \) are the specific energies of transformations at temperatures \( \theta_{\text{Curie}} \) and \( \theta_{\text{top}} \), \( V_0 \) and \( V_i \) values at \( \theta = 0 \) and at infinite, \( \sigma \) - gaussian standard deviation and \( \tau \) - time constant. The model is represented in fig. 4.

![Fig. 4. The model of specific heat variable with temperature during entire process.](image)

**4. THE COVERED CRUCIBLE FURNACE**

For the covered furnace there is in addition a more complicated problem of modeling the thermal exchange between the surface of the metallic bath and the inside surfaces of the lid and the "free" walls of the crucible. The correct analysis of the processes in this volume has to take into account the complexity of thermal transmission phenomena. One of the variants considers in the calculus domain of the thermal problem a corresponding region for the closed air volume inside the furnace. Its thermal conduction properties must be equivalent to convection and radiation within the real system.

There is a natural convection in a closed space. The temperature distribution may be as in fig. 5. In first part of the heating process, when the temperature of the charge is much more higher in the exterior area (thermal sources are concentrated on the skin depth), because of relative large dimensions of the interior of the furnace, the ascending and descending currents flow without interaction. To the end of the process, when temperature of the charge surface is approximatively uniform, the convective currents from the charge surface have a cellular aspect, which is characteristic to the natural convection in the closed spaces (fig. 6 - 7).

For example, determination of convective exchange coefficient between the surfaces of charge and the lid, which are situated at distance \( d \) one from another and determinarea coeficientului de schimb termic convectiv între suprafețele șarjei și capacului, aflate la distanța \( d \) una de alta, întrre care există o diferență de...
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Temperatura $\Delta \theta$, şi care limitează un spaţiu în care se găseşte aer, se efectuează exprimând fluxul de căldură transmis ca un flux termic conductiv:

$$\phi = \lambda_{\text{echiv}} \cdot \frac{A \cdot \Delta \theta}{d} = k \cdot \lambda \cdot \frac{A \cdot \Delta \theta}{d}$$  \hspace{1cm} (25)

where the coefficient

$$k = \frac{\lambda_{\text{echiv}}}{\lambda} = \frac{\alpha_e \cdot d}{\lambda}$$  \hspace{1cm} (26)

depends on Grashof and Prandtl invariants. If $10^2 < (\text{Gr} \cdot \text{Pr}) \leq 10^8$, we have:

$$k = 1 + \frac{m \cdot (\text{Gr} \cdot \text{Pr})_d}{(\text{Gr} \cdot \text{Pr})_d + n}$$  \hspace{1cm} (27)

where $m$, $n$, $r$ are given for different positions and orientation of the surfaces.

![Fig. 5 – Temperature distribution inside of the covered furnace](image)

![Fig. 6 – The accessional convective currents in furnace at the beginning of the process](image)

![Fig. 7 – The cellular convective currents at the end of the process, when the temperature of surface of the bath is aproximately constant](image)
If we want to study the thermal transmission by radiation within the covered furnace, one method consists of electrical analogy. The simplified cylindrical furnace and the equivalent diagram are shown in fig. 8, where $F_{ij}$ are the geometric factors between surfaces 1, 2 and 3. $A_i$, $T_i$ and $\varepsilon_i$ represents the surfaces area, the temperature and respectively the emissivity of the surfaces and $\Phi_i$ the thermal flux. The diagram give 9 equations with 9 unknowns: $\Phi_1 \ldots \Phi_6$, $M_1$, $M_2$, $M_3$. For a given furnace, if we know $A_i$, $\varepsilon_i$, $T_i$, $F_{ij}$, we can solve the system. The temperatures of the inside surfaces, in different points may be determined by micrometer, with total radiation pyrometer and $T_i$ are the medium values.

We can equivala thermal transfer by radiation between the surfaces with a thermal conduction, finding in this way an equivalent (for radiation) thermal conduction of air volume in the interior. For example, thermal transfer between surfaces 1 and 2 we have:

$$\phi = \Phi_5 = \frac{T_1 - T_2}{x} \quad \text{and} \quad \lambda_{\text{equiv}} = \frac{\Phi_5 \cdot x}{(T_1 - T_2) \cdot A}$$

and if we reiterate this algorithm at different moments, we obtain $\lambda_{\text{equiv}}$ variable with temperature.

Fig. 8 – Thermal exchange by radiation within covered furnace – the electric analogy

5. APPLICATION

This application consists of the complex model of a crucible induction furnace CALAMARI type (Italy), from INFRATIREA S.A. Oradea factory. The
furnace melts a charge made by 5 t cast-iron, the melting time being circa 4.5
hours. The geometry of the calculus domain reproduces exactly the real physical
system. The inner side of the furnace crucible is slightly conical with a medium
interior diameter of 860 mm and a total height to the lid, of 1500 mm. At the
bottom the inner diameter of the crucible diminish to 700 mm for a increased
mechanical resistance against stirring. The inner diameter of the inductor is of 1180
mm and the medium thickness of refractory wall is of 160 mm. The lid has
approximatively a segment of crown shape with inner radius of 1040 mm, the wall
thickness of 150 mm and the angle of 50 degree. The inductor has 24 winding
uniform distributed along the total height of 1250 mm. In the model, we considers
that static converter has the nominal voltage $U_{\text{ursa}} = 900$ V at frequency $f = 250$ Hz.
The modeling was performed usind Flux 2D program. The calculus domain has 32
surface regions and 29 lineic regions. The mesh shown in fig. 9 has 75305 nodes,
36558 surface elements and 859 linear elements.

From 20 to 1550 °C, heating time, was 4.5 hours, in a good agreement with
the real process. Induced power in charge varies between 787.1 kW and 384.5 kW.
The medium value of the induced power in charge was 420.1 kW, the medium
value of the Joule losses in the inductor – 135.7 kW and the medium value of the
total thermal losses (inclusively heating the refractory from ambient temperature) –
141.2 kW. Electrical efficiency varies between 0.894 and 0.732 and global
efficiency for entire process was 0.51 (the process starts from “cold” stage).

For the region corresponding at interior free space within the furnace we
consider an equivalent thermal conductivity $\lambda_{\text{echv}} = 5 \cdot \exp(\theta/250) + 5$ W/mK and
specific heat $\gamma_c = 1003 \cdot \exp(- \theta/300) + 290$ J/m$^3$/°C.

Figures 10-11 present the time variation of the induced power in charge and
of the total thermal losses through the exchange surfaces. Fig. 12 shows the
variation of the magnetic relative permeability in charge corresponding at tree
points, at same $z$ coordinate and at different radius $r$. In fig. 13 is represented the
temperature distribution along inductor height, between the insulation of inductor
windings and the thermal insulant layer of the crucible. Fig. 14 shows the
temperature chart in the calculus domain at different moments of the heating
process.

6. CONCLUSIONS

The paper emphasize the specific problems that appear on the modelling of
the electrothermal process within crucible induction furnace. Simplified models
often gives incorrect results. An accurate model in a good agreement with some
experimental data is very useful to study the furnace in order to improve the
operating parameters and to anticipate drawbacks like overheating of different
materials. The application presented exemplifies these particularities.
Fig. 9 – The calculus domain for the magnetic problem of application and the details of adapted mesh.
Fig. 10 – The time variation of the induce power in charge

Fig. 11 – The variation of the total losses trough the thermal exchange surfaces

Fig. 12 – Time variation of permeability of charge in tree points at same \( z \) and different radius \( r \)

Fig. 13 – The distribution of temperature along inductor height at the end of heating
Fig. 14 – The chart of temperature in the calculus domain at different moments during heating process

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