TIME DOMAIN AND FREQUENCY DOMAIN STEADY STATE COMPUTATION OF NONLINEAR CIRCUITS

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The paper presents a comparative study between time domain and frequency domain methods for fast computation of the periodic steady state for nonlinear circuits, from the viewpoint of the efficiency (computation time, convergence, stability and errors). The circuit chosen for proposed analysis is a nonlinear dynamic circuit of second order with stiff behaviour.

1. INTRODUCTION

The problem of the fast computation of the periodic steady state in nonlinear electric circuits has a wide domain of application both in energetics, for the calculation of the electrical networks which work in distortion condition owing to nonlinear consumers (rectifiers, coils with magnetic core, etc.) for the evaluation of the unexpected effects as the power factor reduction, the apparition of the over-voltages and over-currents at resonance on the higher harmonics, etc. and in electronics, for the determination of the amplifier linearity, distortion, power level, transfer characteristic, etc.

The computation of the steady state of nonlinear circuits by the brute-force method (the integration of circuit equations until all transients responses decay) is often very time-consuming [1]. There are three classes of efficient methods to overcome this difficulty: shooting methods, finite difference methods, and harmonic balance methods. The shooting methods are the most efficient ones from the viewpoint of computation time, but the harmonic balance is efficient for strongly nonlinear circuits.

2. TIME DOMAIN METHODS

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The time domain steady state computation for nonlinear dynamic circuits consists in the solving of a differential equations system of first order written in normal form

\[
\begin{align*}
\frac{dx}{dt} = f(x,t), & \quad t \geq t_0, \\
x(0) = x_0
\end{align*}
\]  

(1)

where \( x \in \mathbb{R}^n \) is the state variables vector and \( x(0) = x_0 \) represents the initial condition.

The numerical solving of the system (1) imposes its transforming in a nonlinear algebraic equations system by using of a numerical integration method, like implicit Euler formula

\[
\sum_{k=0}^{k+1} x_{k+1} = x_k + h \cdot f(x_{k+1}, t_{k+1}).
\]

(2)

Integrating the system (1) over a period \( T \) it follows:

\[
x(T) = x(0) + \int_0^T f(x,0,x(0))dt = \Phi(x(0))
\]

(3)

The periodic state is established if the periodicity condition of the circuit solution \( x(T) = x(0) \) is fulfilled, which leads to the next relation

\[
x(0) = \Phi(x(0))
\]

(4)

Therefore the computation of the periodic state solution is reduced to the determination of the initial condition \( x(0) = x_0^* \), which represents the fixed point of the application defined by relation (3). The solution is obtained integrating the system (1) over a period, with the initial condition \( x(0) = x_0^* \).

A shooting method looks for a better initial condition so that transient components are negligible. The solution is obtained by the fixed point determination of the application \( \Phi \) by means of the objective function formulation.

2.1. SHOOTING WITH NEWTON-RAPHSON

The objective function derives from relation (3), for this method

\[
p(x_0) = \Phi(x_0) - x_0 = x(T) - x_0 = 0.
\]

(5)

The fixed point determination of the \( \Phi \) application consists in the solving of the next nonlinear equations

\[
p(x_0) = 0
\]

(6)
The above equation is solved by the Newton-Raphson algorithm [3]

$$x_0^{(i+1)} = x_0^{(i)} - [J_p (x_0^{(i)})]^{-1} \left[ \Phi(x_0^{(i)}) - x_0^{(i)} \right]$$

(7)

where $J_p (x_0^{(i)})$ is Jacoby matrix of the $p$ function, which is calculated in the point $x_0^{(i)}$.

2.2. SHOOTING BY LINEAR EXTRAPOLATION

This method [4] assumes that the state vector at the begining of the $i+1$ period $(x_0^{(i+1)})$ is an affine function of the state vector at the begining of the previous period $x_0^{(i)}$. It follows that the state vector at the begining of the periodic solution $x^*$ is

$$x_0^{(i+1)} = Ax_0^{(i)} + b \quad x^* = Ax^* + b \Rightarrow x^* (1 - A) = b$$

(8)

where $A$ and $b$ are unknown.

The fixed point of the affine application is calculated with the relation

$$x_0^{(i+1)} = x_0^* = \frac{\sum_{i=0}^{p} c_i x_0^{(i)}}{\sum_{i=0}^{p} c_i}$$

(9)

where $c_p = -1$ and the others coefficients result from the next relation

$$\begin{bmatrix} \Delta x_0^0 \\ \Delta x_0^1 \\ \vdots \\ \Delta x_0^{p-1} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{p-1} \end{bmatrix}$$

(10)

in which $\Delta x_0^{(i)} = x_0^{(i+1)} - x_0^{(i)}$.

3. FREQUENCY DOMAIN METHODS

The harmonic balance is a frequency domain method [1], which represents the solution as a linear combination of some periodic functions from an ortonormal basis. The periodic steady state computation consists in the calculation of the functions coefficients.
The used functions are the trigonometric ones: $\phi_{1k} = \cos(k\omega t)$ and $\phi_{2k} = \sin(k\omega t)$ or $\phi_k = e^{jka}$, $k = 0,1,...,H - 1$, where $H$ represents the harmonics number which is taken into account.

Using the nodal modified method for a nonlinear circuit which contains only voltage controlled elements, the following equations are obtained:

$$f(u,t) = \dot{q}(u(t)) + i(u(t)) + \int_{-\infty}^{\infty} y(t - \tau)u(\tau)d\tau + i_s(t) = 0$$  \hspace{1cm} (11)

where $\dot{q}(u(t))$ represent the currents through nonlinear capacitors, $i(u(t))$ are the currents through nonlinear resistors and through voltage controlled sources, $y(t)$ represents the impulse response of linear sub-circuit and $i_s(t)$ are the independent sources contribution.

Applying the Fourier transformer to relation (11) it follows:

$$F(U) = I(U) + j\Omega Q(U) + YU + I_s$$  \hspace{1cm} (12)

The nonlinear equation (12) can be solved using Newton-Raphson algorithm:

$$U^{(i+1)} = U^{(i)} - J^{-1}_F(U^{(i)}) \cdot F(U^{(i)})$$  \hspace{1cm} (13)

where $J_F$ has $(2H-1)N^* (2H-1)N$ dimension and represents harmonic Jacoby matrix.

4. SIMULATION RESULTS

The circuit in Fig. 1[2] is used to compare the shooting and harmonic balance methods. This is a second order nonlinear dynamic circuit which has one time constant much greater than the excitation period. Therefore the transient behavior is very long and the brute-force method requires a very long computation time. The circuit nonlinearity is given by a diode which is modelled by a piecewise linear characteristic (Fig. 2.).

The circuit parameters are: $L_1=1 \text{ mH}$, $L_2=100 \text{ H}$, $R=10 \Omega$. The diode model parameters are: $I_{KF}=20 \text{ A}$, $I_s=1nA$, $R_s=1\mu\Omega$, $V_j=0.6V$, $R_d=0.1m\Omega$. The voltage source has two components: a sinusoidal one of 1 kHz with the amplitude of 5 V and a dc one of 5 V (Fig. 3).

The circuit operation can be described as: as long as the diode is on the $L_2$ inductances discharges with a very long time constant ($10^6$s) with respect to the excitation period (Fig. 4.); when the diode is off, the two inductors can be considered in series and current is practically the same if we neglect the reverse diode current.
The unknown vector contains the state variables, namely the currents through the two coils \( x = [i_1, i_2]^T \).

For the comparative study of the shooting methods the next testing conditions are chosen:
- for the initial condition: \( (i_1(0), i_2(0)) = [(0,0), (0,100)] \) A;
- for the data accuracy: 10 significant digits with imposed error \( \varepsilon = 10^{-5} \), respectively 20 significant digits with imposed error \( \varepsilon = 10^{-10} \);
- for the time step: \( h = \frac{T}{100}, \frac{T}{1000} \).

The results are given in the Table 1, where \( m \) is the point number of the discrete network, \( NI \) is the iteration number after that the periodic steady state is reached, \( NP \) represents the period number performed by the numerical integration algorithm and \( TC \) is computation time in seconds. Also, the following notations are used: SNR for shooting method with Newton Raphson, SLE for shooting method...
by linear extrapolation and SEE for shooting method by exponential extrapolation with a time constant.

Table 1
The results of the simulation.

<table>
<thead>
<tr>
<th>(i_{10},i_{20}) [A]</th>
<th>Digits</th>
<th>m</th>
<th>SNR</th>
<th>SLE</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>NI</td>
<td>NP</td>
<td>TC [s]</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>(0,0)</td>
<td>10</td>
<td>100</td>
<td>4</td>
<td>4</td>
<td>0,5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>4</td>
<td>4</td>
<td>11,2</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>100</td>
<td>6</td>
<td>6</td>
<td>0,8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>7</td>
<td>7</td>
<td>20,3</td>
</tr>
<tr>
<td>(0,100)</td>
<td>10</td>
<td>100</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>100</td>
<td>7</td>
<td>7</td>
<td>1,1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>8</td>
<td>8</td>
<td>26,2</td>
</tr>
</tbody>
</table>

The program codes are written in MAPLE language and where the algorithm is not convergent to the accurate solution the * character is written.

For harmonic balance, the periodic steady state solution is obtained with APLAC simulator. A correct solution is obtained only if the harmonic number is greater than 40. In table 2 is given the computation time for different values of harmonic number.

Table 2
The simulation results.

<table>
<thead>
<tr>
<th>Harmonic number</th>
<th>4</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
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</thead>
<tbody>
<tr>
<td>TC [s]</td>
<td>0,07</td>
<td>0,09</td>
<td>0,131</td>
<td>14,83</td>
<td>37,17</td>
<td>95</td>
</tr>
</tbody>
</table>

In figure 5 are presented the circuit responses for 4 harmonics (fig. 5 a)) and for 50 harmonics (fig. 5 b)).
4. CONCLUSIONS

From the Tables 1 and 2 it follows:
- shooting method with Newton Raphson is the fastest one, but it has convergence problems if the initial conditions are far off from the periodic state values;
- for the low accuracy of the data the shooting method by linear extrapolation are not convergent;
- for the high accuracy of the data all methods are convergent;
- for an accurate solution the computation time is greater for the harmonic balance than for the shooting methods.

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